Teaching geometry to students (from five to eight years old)

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For students from 5 to 8 years old, the spatial and geometrical learning concern mainly the control of its relations to real space (tracking, travelling), the recognition of objects and shapes, and their representation by drawings using straight lines. The ERMEL team, in France, is experimenting teaching situations on space and geometric learning in the 2nd cycle of primary education (5–8 years old student) and is building complete engineering to teach math in elementary school. Our methodology involves an analysis of the student's way to solve problems and thus their abilities. Experiments conducted in a great number of classrooms allow us to better understand the components of this learning in terms of knowledge and abilities. Thanks to these results, our hypothesis and choices have evolved.

Keywords: Geometry, teaching, learning, primary education, spatial problems solving.

RESEARCH'S PRESENTATION

Issue of this paper

Our research takes place in the French context of geometry teaching in primary school. Our goal is to build proven, complete and reliable teaching engineering and thus to improve geometry teaching.

By submitting our results and remaining questions we want to contribute to the working group's thought. In a European context we also want to compare our approach to others and seek our results significance.

Why working on geometry teaching?

For a long time, Berthelot and Salin (1992, 1993), highlighted the difficulties of geometry teaching in France:

> No one disputes, for example the usefulness of knowing how to do a multiplication, so learning it in school, has been normal for a century or so. [...] For the geometrical knowledge, beliefs are less assertive, [...]. So they feel "authorized" to take leeway with the school curricula, that is to say, to ignore this part (as well as high school teachers ignore 3D geometry) (translated from (Berthelot & Salin, 1993, p. 39))

An analysis of teachers' representation about geometry teaching (in process) confirms that in 2014, things have not really changed. This work, based on interviews, also shows that both geometrical and didactics knowledge of teachers (even experienced) is weak.

As far as geometry is concerned, textbooks mainly focus on students' work, on drawings and being able to write the proper name on the proper drawing.

Geometry learning is not seen as a social necessity by the families and sometimes by teachers. Moreover, its contribution to subsequent learning is often reduced in the classrooms at an early learning, even ineffective, of geometrical vocabulary.

This research is also based on the idea that students' abilities are insufficiently taken into account in geometry teaching in primary school. Thus we have to identify the knowledge at stake in this learning and take former students' knowledge into account.

By starting our research from the analysis of students' abilities and geometric concepts we develop an uncommon methodology (described below).

About us

ERMEL is a research team on mathematics education in primary school (in French "Équipe de Recherche en Mathématiques à l'École Élémentaire"), which belongs to the French institute of education (IFé). This team is made up of primary school teachers, teachers' trainers and researchers working in different cities of France (Châlons-en-Champagne, Grenoble, Lyon, Paris...).

The ERMEL team conducted studies on teaching and learning of number system and arithmetic's form 1985 to 1998 and since 1998 on geometry teaching and learning. Results of theses researches lead to comprehensive book publications on teaching engineering (Équipe de Recherche en Mathématiques à l'École Élémentaire [ERMEL], 1998, 1999 & 2006).

The aim of the current research is to analyse spatial and geometric skills that 5 to 8 years old students can build.

Methodology

Our research has been and is sometimes called "action research" in the French tradition. We do not deny that term in its meaning and use Hugon and Seibel's (1988): "Research in which there is a deliberate transformation of reality; research that has a dual purpose: to transform reality and produce knowledge about these changes." Our research actually intended to produce resources that are analysing issues of education in relation with school curricula and their changes.

On the other hand, our methodology is quite different form the English tradition of "action research in education" as defined by Sagor (2000, p. 3):

> It is a disciplined process of inquiry conducted by and for those taking the action. The primary reason for engaging in action research is to assist the "actor" in improving and/or refining his or her actions. Practitioners who engage in action research inevitably find it to be an empowering experience.

Even if teachers and teachers training have an important place in our research, we define our methodology as a didactic engineering. Our approach has in common with the didactic engineering concept the general questions it allows to address. In fact, the term "didactic engineering" appears in the mathematics teaching in France in the early 80s as a way to answer two fundamental questions translated from Chevallard (1982 as cited in Artigue, 2002, p.59):

> How to take into account the complexity of the classroom in research methodology?
>
> How to think about the relationship between research and action on the education system? [...] As a research methodology, didactic engineering is different from the usual experimental methods by its validation mode. This internal validation method is based on the confrontation between an a priori analysis in which are engaged a number of assumptions and a post hoc analysis that relies on data from the actual implementation.

The ERMEL research team develops an analysis of students' knowledge, issues of teaching, and offers a didactic engineering based on an experiment conducted in many classrooms for several years.

This research is derived from the analysis of the challenges of teaching mathematics in the field. Once the needs are identified, such research involves several steps:

1) An analysis of the mathematical knowledge (problems, properties...) at stake, as well as students' knowledge and abilities;

2) An explanation of educational issues and the organisation of the study of the different notions throughout the years;

3) Development of teaching situations and tests in several classrooms.

These last three components interact: the identification of students' abilities is the outcome of experiments conducted.

4) Writing a book for teachers and trainers with an explanation of the issues of learning and teaching, a description of the identified learning situations, a reasoned choice of learning roadmap and syllabus planning. These books are often references in training in France.

The diagram below (Diagram 1) illustrates our methodology.

Analysing teaching and learning process require theoretical frameworks.

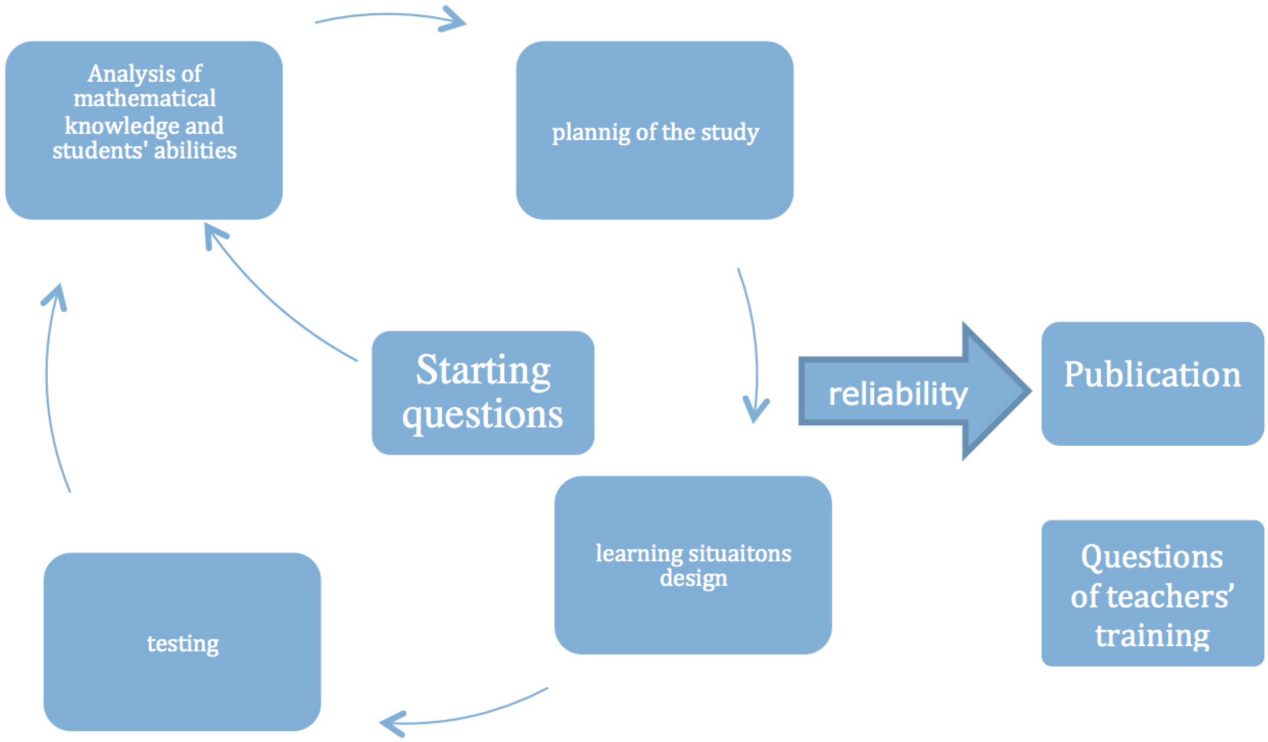

Diagram 1: Steps of ERMEL's methodology

Theoretical frameworks

We mainly focus on a framework that is preeminent in the French context, the Theory of Didactical Situations based on the ideas below:

> Mathematicians don't communicate their results in the form in which they discover them; they re-organize them, they give them as general a form as possible. Mathematicians perform a "didactical practice" which consists of putting knowledge into a communicable, decontextualized, depersonalized, detemporalized form.
>
> The teacher first undertakes the opposite action; a recontextualization and a repersonalization of knowledge. She looks for situations which can give meaning to the knowledge to be taught. But when the student has answered to the proposed situation (...) she will have to redepersonalize and redecontextualize, with the assistance of the teacher, the knowledge she has produced so that she can see that it has a universal character, and that it is re-usable cultural knowledge. (Brousseau, 1997, p. 227)

For teachers, building learning-situations is building a problem where the knowledge is recontextualized. By solving this problem, the student will acquire the knowledge at stake. There are different kinds of problems corresponding to different ways to reach the knowledge (more or less efficiently).

As far as geometry is concerned, we also use concepts highlighted by former researches. We distinguish between spatial knowledge and geometrical knowledge:

> In general we can distinguish between two kinds of problems:
>
> — spatial problems thus characterized: their purpose concerns the sensitive world; they can focus on implementing actions: making, shifting, moving, drawing, etc. communication about actions or findings [...] The success or failure is determined by the subject itself by comparing the expected result and the result;
>
> — geometric problems, like that word is used in mathematics: Solving a geometrical problem is an activity involving the necessary and non-adversarial nature of certain properties of the geometrical objects" (translated from Berthelot & Salin, 1993, p. 41).

We also use the work of these authors, which defines different kinds of space in which spatial problems can be placed: micro space (very close to the subject, object can be moved, touched, turned) meso-space (surrounding space, between the arms of the subject, he can have a comprehensive view, he can move in the space) and macro space (far space, the view is more local, subject has to conceptualize). The space defined by the sheet of paper can be called graphical space and has also special features. Computer screen also form a new type of space depending on software uses (for example dynamical geometry software). Theses spaces are so many choices and so many parameters of the learning situation.

We focus on the alignment and straightness concept to make our approach explicit.

ALIGNMENT AND STRAIGHTNESS IN THE 2ND CYCLE OF EDUCATION

(2nd cycle of French school is 1st and 2nd grade in the USA)

This research was attributable mainly to the fact that the teaching of geometry in the primary grades does not adequately take into account the knowledge children can develop when solving problems. This research therefore requires identifying the skills involved in these learnings and take into account the initial knowledge in these fields.

As often with geometric concepts introduced in elementary school, the concept of straight line has a double aspect:

- it allows to represent real-world objects or actions

- it is a component of a geometric knowledge constitution that has properties that students gradually discover: in this case it is the properties of the straight line and the constraints of its drawing.

This learning raises several questions: in the 2nd cycle, what are the possible links between these two aspects (alignment of points and straightness of lines)? In particular can the procedures developed in the meso-space be reused on the paper? Is it better to start with experiences in the meso-space to show a straight line as a solution of an alignment problem?

Meanings of the straight line

The notion of straight line can be understood by the 2nd cycle's student through different meanings induced by perception or experience, which can be used in problems for:

1) The graphic representation of physical objects soliciting properties of the straight line, including:

— a material object: a stretched wire, a straight edge object, a light ray, the fold of a paper ...

— a border between material objects :

 — two planar regions of space, as the edges of a polyhedron;

 — two regions of the map, as the sides of a polygon, of half-planes ;

— a subject of the graphics world : a straight line (which may be extended beyond the ends);

— a trace :

 — a print trace produced by a path with the ruler ;

 — a print screen trace caused by the “straight” tool in a dynamic geometry software (like Cabri Geometry)...;

— the path of a rectilinear object (ball ...).

For all these problems in connection with these first meanings, the points do not play an important role.

2) a set of aligned points : the locus of points aligned with two points, for instance for sight problems in the meso-space.

Properties

Our previous work research (ERMEL, 2006) on geometric learning has confirmed that the properties attributed to the straight line by the students aged 8 to 9 (CE2), are limited to those related to the perception of lines drawn. From a theoretical point of view, if a straight line can be characterized in various ways, as the effect (invariance) of a transformation, as the intersection of planes, for the student of six or seven years old the concept of the direction (extension) is the first they meet. In fact for students of this age a straight line is simply the straight line drawn on a sheet. Several properties of the line, as a mathematical object (in particular the fact that the line is formed by an infinite set of points) are not accessible to the primary school.

Questions

One of the challenges of our research on 2nd cycle was for us to determine:

— What is the initial knowledge of the students? What perceptions, what experience do they have of straight lines?

— Among the meanings of straight line which must be preferred?

Initial hypothesis

We therefore sought to clarify the meanings of straight lines that may be encountered or that are accessible to 2nd cycle students and to create problems, allowing a passage to geometric knowledge.

We thought that the fact that a line can be extended for solving an alignment problem should be learned.

At the beginning of this research, we considered teaching a grasp of the concept of alignment through the experiment centred on the idea of hiding an object (using the sight) in the meso-space, then again on paper in order to highlight the utility of using the straight line.

We proposed a problem ("Plots") that takes place in the schoolyard (Figure 1): students have to find locations (using the sight) where an object hides another one (I see the red plot hiding the green one and the yellow plot hiding the orange one). The problem was after proposed on the paper sheet (representing a top view of the yard). In solving the problem on paper the students were not using straight lines to find the locations; broken lines have been uses to depict alignments.

These experiments revealed activity transition-related difficulties conducted in the meso-space activities on paper.

First experimental results for CP (1st grade US – Year 2 GB) and CE1 (2nd grade US – Year 3 GB)

Students had developed spatial resolution procedures based on the sight and other gestures in the meso-space. But the modelling by a straight line was not effective in solving a similar situation on paper. In addition there were difficulties for drawing straight lines with a ruler, many productions included broken lines and not straight lines to represent the target.

Analysis of the difficulties in the transition from meso-space to micro-space

They seem related to:

1) An understanding of modelling on a sheet of a situation experienced in the schoolyard:

— Meso 3D / micro 2D (top view ...) and the disappearance of the subject (student) in the micro-space device.

— A representation of physical objects by schematizations (circles, dots).

2) What the straight line was supposed to represent.

3) For the drawing of plots straight lines.

The use of dynamic geometry software (DGS) seemed to enable this switch in CE1 (7-years-old): students using the straight lines to produce with DGS a solution to the alignment problem. However, we have not been


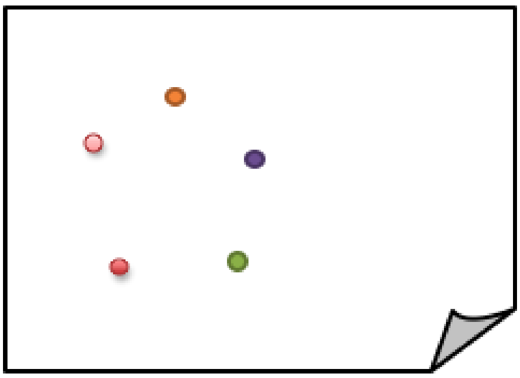

Figure 1: "Plot" situation in the schoolyard (left) and in sheet of paper (right)

able to check the reinvestment in the longer term on the sheet of paper.

The analysis of these problems (items 2 and 3 above) also highlighted the need for a specific work to allow the apprehension of the meanings and some properties right from the 2nd cycle (the term « straightness ») relatively independent of the alignment point problems. This corresponds to the first set of meanings of the straight line as shown on page 5.

Rolet (2003, p. 7) who also analysed 8-year-old students' work in different spaces noticed difficulties: "[students] they took the piece of rope not like a part of a line but like a segment. [...] Cabri-geometry was also a good instrumented space to give a real status to concepts like points: 'visible' vs defined, 'draggable' (or independent) vs fixed (or dependent). But understanding of these concepts still presents many difficulties for all students."

SPECIFIC LEARNING OF STRAIGHTNESS

Meanings of the concept of straight line

The meanings of the straight line can be associated with problems concerning either: the production of straight lines, the identification of straight lines (judgment), the choice and use of instruments that could be used to identify or produce an alignment or a straight line.

Among the meanings of a straight line, we chose to emphasize, for 6-year-old students, those of edge parallel strips, contour of a geometric figure. Validation criteria are either simply perceptual (regularity of the trace or resemblance to a model) or related to a more or less explicit reference to parallelism (direction ...) or consist of a practical validation (coincidence ends, overlay actions objects ...).

Presentation of a situation and common Goals

Students are working on "superpositioned form". They have to do a stack of shapes (rectangles, triangles) and draw their superposition.

These meanings can be associated to problems concerning either:

— the production of straight lines;

— the identification of straight lines (judgment);

— the recognition and use of instruments that could be used to identify where to produce an alignment or a straight line.

This work on the notion of " straightness", although it can be started in kindergarten (Year 1 in GB, called "grande section" in France) with the production of regular lines made in drawing activities, really takes its geometric dimension at 1st grade (6-years-old, CP in France) where the properties of straight line can therefore be apprehended. These properties are not yet objects of study for themselves, but are first experiences:

— a line can represent something that does not leave a physical form (e.g. sight);

— a drawn line may be associated with instruments;

— a line may be extended, for example to represent a hidden object.

Different levels of understanding control may be associated with these meanings:

— understand the need to draw a straight line to solve the problem; this is the role of formulations (validation through language, rather consensual), role of gestures (hands);

— learn how to place the ruler to draw a longer line; it is the knowledge of the method and the technological aspect: name tools, describe the action...

— mastery of drawing (validation by production).

ASSESSMENT FINDINGS AND QUESTIONS

Robustness/reliability of situations

— Devices (progressions, situations) that we have developed favour a knowledge construction based on problem solving. They have a certain "robustness" due in particular to the fact that the results and procedures that will be produced by students in a class are described in the description of situations, allowing the teacher, in general non-specialist in mathematics, to anticipate their decisions based on its own class productions. This reliability seems partly due to the coherence between the concepts of learning and the proposed situations and, secondly, to their experimentation in many classes for many years.

One of the challenges of our methodology is to allow ownership by teachers of educational created devices.

The conditions that we consider necessary for the appropriation of our devices by those teachers are:

— a better perception of the relationship between spatial knowledge and geometrical knowledge that students can develop;

— an awareness of all the meanings of a concept and situations associated;

— the identification of the essential characteristics of teaching situations;

For this we highlight these elements in our publications.

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